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ABSTRACT

This study was designed to determine if there were certain aspects of cognitive/personality style that affected prospective teachers' choice of problem-solving paradigm and presentation of inquiry. Fifty-six undergraduate elementary education majors were given the Paragraph Completion Test, which measures conceptual level, and the Group Embedded Figures Test, which measures field-dependence-independence, along with a self-report instrument designed to measure choice of problem-solving method and presentation format. Results suggest prospective teachers' choice of problem-solving paradigm is affected by their conceptual level and field-dependence-independence. (Author)

Perspectives on Problem Solving: Person versus Paradigm

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Our view of human being as learner has changed over time and at present includes a perspective which consists of "needs for excitement, novelty, sensory variation, and perhaps above all the challenge of the problematic (Getzels, 1974, p. 10)." Of the numerous questions that we can raise concerning the individual's meeting of this challenge is the question of whether problem-solving approaches are organism dominated or stimulus dominated. That is to say, do particular individuals with specific cognitive/personality profiles approach problems in specific ways regardless of the nature and/or difficulty of the problem, or rather do particular classes of problems by their nature suggest specific algorithms or approaches? If it is that the problem-solving approaches are determined by the person, two questions which follow are: what specific approaches does an individual choose and what particular aspects of the individual suggest these approaches? If it is the latter that the problem approach is determined by the nature of the problem, we wonder in what ways the approaches to the problematic can be presented to be most effective in improving the individual's problem-solving ability and capacity?

If individuals determine their own approach to problem solving as a function of the cognitive/personality lenses through which they view the problematic, then it would be valuable to locate the ways in which particular individuals construct knowledge. This will provide us in education with a sense of where a particular individual perspective is grounded and we can work toward engaging him/her in a personally responsive way and help this person become more adept at responding critically to diverse problematic situations. On the other hand, if particular classes of problems suggest their own problem-solving model, then we should make efforts to classify and group problems which share common investigative approaches in order to help individuals organize the world they experience via the problematic which surrounds them.

The question of whether the situation or the person provides the lens for analysis is of vital concern to teacher education. If it is strongly suggested that problems rather than people shape the perspective, then we in teacher education need to make efforts in at least two directions: one is to explore ways in which particular approaches are structured and determine how and in what ways these structures are or are not confluent with other problem-solving models; and second, to help prospective teachers understand the dynamics of the structures and the means by which structures are connected for a great deal of learning comes from reasoning by comparison and analogy. Whereas if the individual provides the lens, then educators need to consider at least two additional perspectives: one is to begin to determine the particular lenses that are used by a teacher as well as how these lenses affect her/his presentations of problem situations and solutions; and second, is to make value judgements and decisions as to which models of inquiry appear to be most productive and determine ways in which prospective teachers can be redirected, if necessary, so as to develop more powerful investigative techniques.

Some argue that "the development of the ability to solve problems is probably the most important aspect of one's education (Troutman & Lichtenberg, 1974, p. 590)." In this regard we need to consider what, if any, effects manifested in teacher planning and presentation are stimulated by a problem's complexity for conceivably "different instructional procedures could activate different aspects of existing cognitive structures (Mayer & Greeno, 1972, p. 165)" in students.

Problem Solving in Mathematics

These concerns likewise have specific value in the study of mathematics as problem solving is regarded as the "basic mathematical activity (Rosenbloom, 1966, p. 130)." A reading of mathematics curricula and research findings suggest that certain problem-solving models are offered to students as effective procedures for constructing mathematical knowledge. The models may be separated into two groups and I will refer to one group as the Polya method and the other as the Dahmus method.

Polya's (1954,1957) method emphasizes the heuristic and provides a perspective of problem solving that is both economical and effective in computer performance (Kilpatrick, 1969). This method emphasizes understanding and directs the student to consider the problem in terms of related problems, to consider specific cases of the problem, to guess, and to locate the problem in a more general context. Variations of this approach are suggested by other mathematics educators (Thompson, 1976; Post & Brennan, 1976). Dahmus' (1970) method differs markedly from Polya's method in that it suggests a translation process in which each verbal statement is translated into a corresponding mathematical statement. Here the emphasis is grounded in the particular problem and operationalizes a procedure that is similar to constructing a jig-saw puzzle, piece by piece, without the necessity of considering the whole.

Research in the area of problem solving in general, and in using specific techniques in particular, has not produced a clear direction for teaching problem solving in the mathematic classroom. Articles dealing with the state of the art of problem solving up to less than a decade ago reflect conflicting results (Gorman, 1967; Suydam, 1967; Kilpatrick, 1969). The complexity of the situation is suggested, for example, by the research of Stillwell (1967) who found that mathematics teachers gave only 3% of class time to duscussion of methods for problem solving and only 7% to reflection on the procedures and implications of problem-solving techniques. In the past half-dozen years, studies which examined the different effects of different problem-solving techniques have not been successful in determining that any particular approach was, in general, more effective than any other (Post, 1968; Bassler, et al., 1975; Post & Brennan, 1976). One does find, however, a change in consciousness regarding the direction of emphasis in the problem solving area from the process to the person as expressed in the change in views found in the 21st and 33rd Yearbooks of the National Council of Teachers of Mathematics. The 21st Yearbook, published in 1953, presents an article on problem solving in which the authors suggest a unique process for solving mathematics problems (Henderson & Pingry, p. 233). However, the article on problem solving in the 33rd Yearbook, published in 1970, offers the conflicting opinion that "teachers should encourage questioning from the students, reward different ways of solving the same problem, and use praise and recognition to encourage explanations of processes and concepts (Kinsella, p. 253)."

The perspective that personality/cognitive factors exert an influence on the educational environment is a relatively new one in general and a particularly new one in problem solving in mathematics. Recent research has suggested that specific aptitudes might well be responsive to particular educational environments. In particular, the research of Hunt (1970; and Tomlinson, 1971), and Witkin (1962, 1973) have demonstrated that conceptual level and field-dependence-independence are personality/cognitive traits which relate to specific learning environments. Hunt and Joyce (1967) found that prospective teachers teaching style was related to their conceptual level: those teachers who had a low conceptual level were not able to radiate a reflective educational environment in which questioning and hypothesizing were encouraged, whereas those teachers who had a high conceptual level were able to radiate such an environment. Witkin & Moore (1974) in reviewing the research on field-dependence-independence found that field-independent people are more analytically oriented than field-dependant people.

With respect to teaching mathematics, there is some additional evidence confirming conceptual level and field-dependence as important determinants. Bien (1974) found that field-dependent children increased their problem solving success when presented with cognitive structuring techniques. And this investigator (Gordon, 1977) found that prospective mathematics teachers with low conceptual level, in general, presented a rule prior to presenting the examples in their lessons, while prospective teachers with high conceptual level tended to present examples prior to the rule, suggesting a high need for structure by those with low conceptual level and a low need for structure by those with high conceptual level. This inverse

relationship between conceptual level and need for structure is in agreement with Hunt's thesis.

Inasmuch as conceptual level is primarily a measure of cognitive complexity and field-dependence-independence a measure of perceptual differentiation, it was anticipated that viewing a crucial aspect of mathematics teaching; namely, problem solving, through both lenses would be of value in gaining a clearer understanding of the stimulus/organism question and whether or not these lenses play an important role in determining teacher presentation of problem-solving techniques.

Method

Fifty-six elementary education majors, mostly seniors, who were students in the undergraduate course, Methods of Teaching Mathematics and Science in the Elementary School, were the subjects in this study.

The prospective teacher's conceptual level was assessed by the Paragraph Completion Test (Hunt, et al., 1973). Subjects were asked to write at least three sentences for each of six topics; e.g., "What I think about rules . . .," which were scored according to Hunt's criteria for measuring need for structure. The complete response to each topic sentence received a score of from 0 to 2.5, depending upon the degree of conceptual complexity and/or interpersonal maturity. The average of the three highest scores indicates the conceptual level (CL) of the respondent. The prospective teacher's field-dependence was assessed by the Group Embedded Figures Test (Oltman, et al,, 1971). Subjects were asked to locate a figure that is embedded within a field for eighteen different figures. The greater the number of figures correctly disembedded from within a field the higher the degree of field-independence. Additionally, the subjects were asked to write five sentences which would express how they would teach each of two mathematics verbal problems--simple and one more complex--and then write their own general procedure as to how to teach children to solve mathematics verbal problems. The simple verbal problem was: "Billy walked three-tenths of a mile on Monday, five-tenths of a mile on Wednesday, and eight-tenths of a mile on Friday. How far did Billy walk?" The more complex verbal problem was: "Mr. Jones had a fence around his farm that was a mile wide by a mile long. Mr. Rivera had divided his farm into two parts. Each part was fenced in and was a mile long by a half-mile wide. Who had more fence? And how much more?"

The following hypotheses were tested:

1. Prospective teachers who either were field-independent or had a high CL would choose the Polya method as their general approach to solving verbal problems while prospective teachers who either were field-dependent or had a low CL would choose the Dahmus method as their general approach.
2. Prospective teachers who chose the Polya method would ask more questions than prospective teachers who chose the Dahmus method.

Results

To test the first hypothesis, conceptual level scores were partitioned into categories of Low, Medium, and High, determined by scores of 0-1.4, 1.5-1.9, and 2 and above (Hunt, 1971). The field-dependence scores were partitioned into three categories: relatively field-dependent (1-9), intermediate (10-13), and relatively field-independent (14-18). From the subjects' general approach to solving mathematical verbal problems the approach was classified into the categories of Polya or Dahmus, depending upon their concern for understanding or translating the problem, respectively. Chi-square analyses suggest, at .05 levels of significance, that prospective teachers who are relatively field-independent or who have a high conceptual level tend to choose the Polya method while relatively field-dependent or low CL teachers tend to choose the Dahmus method (Table 1).

TABLE 1

CHI-SQUARE ANALYSES OF CONCEPTUAL LEVEL AND FIELD-DEPENDENCE WITH CHOICE OF PROBLEM-SOLVING APPROACH

Conceptual Level

	0-1.4	1.5-1.9	2+	
Polya	3	10	21	34
Dahmus	5	11	6	22
	8	21	27	56

$x^2 = 6.62$, df = 2
$p < .05$

Field-Dependence

	1-9	10-13	14-18	
Polya	8	7	19	34
Dahmus	12	4	6	22
	20	11	25	56

$x^2 = 6.08$, df = 2
$p < .05$

Using Chi-Square Analyses with Yates' correction, the second hypothesis was tested for both presentations. The results suggest, at .025 and .001 levels of significance that prospective teachers who chose the Polya method of problem solving asked more questions in both presentations than those who chose the Dahmus method. (It should be noted that there was a decrease in question frequency between the first and second verbal problems; this may be due to the increased complexity of the second as compared to the first verbal problem.)

TABLE 2

CHI-SQUARE ANALYSES OF PROBLEM-SOLVING APPROACH WITH QUESTION FREQUENCY

Verbal Problem #1 (Simple)

question frequency

	none	at least one	
Polya	3	31	34
Dahmus	9	13	22
	12	44	56

$x^2 = 6.39$, df = 1
$p < .025$

Verbal Problem #2 (Complex)

question frequency

	none	at least one	
Polya	5	29	34
Dahmus	16	6	22
	21	35	56

$x^2 = 16.78$, df = 1
$p < .001$

Implications

The results suggest that the cognitive/personality variables of conceptual level and field-dependence-independence are related to choice of problem-solving method; i.e., it seems to be that problem-solving methods are organism dominated rather than stimulus dominated. The choice and presentation of the Dahmus method suggests that these prospective teachers prefer (or compensate by choosing) this method because it does not emphasize understanding of the problem, and as such implies a more passive role in constructing mathematical knowledge. Whereas the choice and presentation of the Polya method suggests an active emphasis in approaching problem solving and as such implies a greater degree of comfort and capability in constructing mathematical knowledge.

Teacher education programs concerned with the teaching of mathematics should make prospective teachers aware of both the Polya and Dahmus methods for solving verbal problems. It seems likely that prospective teachers who would choose the Polya method will have an easier time learning the Dahmus method, than vice versa. The choice of the Dahmus method as a teaching approach is seemingly limited inasmuch as the choice implies that a narrow set of existing knowledge is sufficient for problem solution. Thus, in the context of the teacher as learner model, greater emphasis should be given to solving many and diverse problems for those who would choose the Dahmus method as some research (Clark & Peterson, 1976) on teacher training programs suggests that "the teachers rarely changed their strategy from what they had planned even if instruction was going poorly (p. 11)." In this regard, we need to consider how a particular individual's conceptual level can be raised. Hunt (1971) suggests matching teachers and students so that the teacher's conceptual level is one stage above the student's; in this way, the student's conceptual level will be raised as the interaction continues. This investigator would agree with those who believe "that a prospective teacher should be assigned, when it is feasible, to that training experience that is likely to be most effective for him (Borich & Godbout, 1974,p.4)."

The present findings of a decrease in questioning moves accompanying an increase in problem complexity suggest additional considerations. Inasmuch as the Polya method emphasizes continued student-teacher interaction and encourages consideration of a variety of approaches to come to understand a problem, a decrease in questioning would tend to reduce the effectiveness of this approach. Thus, while teacher education programs should emphasize "a personalized approach", a transcendent emphasis should be given to interactive inquiry. For example, since the research on conceptual level strongly suggests that low conceptual students profit from a highly structured environment, I would suggest that our efforts to provide the needed structure be directed toward developing an inquiry format where the questioning process would emphasize a greater frequency of questions of a limited nature. In this way prospective teachers may come to value the inquiry process and possibly adapt this approach in their teaching style regardless of their conceptual level or degree of field-dependence.

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